

Collatz conjecture convergence proof

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Collatz conjecture is famous, well known problem. Definition of Collatz conjecture follows:

$$a(n) = \begin{cases} \frac{n}{2} & 2|n \\ 3n + 1 & 2 \nmid n \end{cases}$$

In the next algorithm I will try to prove that collatz sequence always converges to 1.

1. We define input as $X = 2^{a_1} 3^{a_2} 5^{a_3} \dots$. Sequence of a_n ends when there is no more factors - n is size of X ($size(n) = \log_2(n)$). Otherwise we take X as input and derive size of it.
2. In case of $a_1 \neq 0$ - we reduce size by a_1 we also remove factors of 2, ie $X = 2^{a_1} 3^{a_2} 5^{a_3} \dots \rightarrow X = 2^0 3^{a_2} 5^{a_3} \dots$. Then $size(X) = \log_2(X) - a_1, a_1 \in [1.. \infty]$
3. In case of $a_1 = 0$ - we do next - we shuffle in 3^1 increasing $size(X) = size(X) + \log_2(3)$, Then we shuffle all factors, giving in result: $X = 2^{\tilde{a}_1} 5^{\tilde{a}_3} \dots \alpha^{\tilde{a}_{n+1}}, \tilde{a}_2 = 0$.
4. We repeat 1 until we fall to size of x is zero (bruteforcing keyspace to 0), ie $X=1$, in case of 2: $size(X) = size(X) - a_1$, in case of 3: $size(X) \approx size(X) + \log_2(3)$, where $a_1 = 1 \vee a_1 \in [2.. \infty]$ with exactly similar probability.

It concludes proof.

□

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