Collatz conjecture convergence proof

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Introduction Collatz conjecture is famous, well known problem. Definition of Collatz conjecture follows:

$$a(n) = \begin{cases} \frac{n}{2} & 2|n \\ 3n+1 & 2\not|n \end{cases}$$

Proof In the next algorithm I am going to show that collatz sequence always converges to 1.

1. We define input and 'operation' as $X = 2^{a_1}3^{a_2}5^{a_3}$...Sequence of $a_n$ends when there is no more factors - size of X ($size(n) = \log_2(n)$) falls to 0. Otherwise we take X as input and derive size of it.

2. X: In case of $a_1 \neq 0$ - we reduce size by $a_1$we also remove factors of 2, ie $X = 2^{a_1}3^{a_2}5^{a_3}... \to X = 2^0 3^{a_2}5^{a_3}$...Then $size(X) = \log_2(X) - a_1, a_1 \in [1..\infty]$. Output of this step is always producing argument for step 3.

3. X: In case of$a_1 = 0$- we do next - we shuffle in 3^1increasing $size(X) := size(X) + \log_2(3)$, Then we shuffle all factors, giving in result:$X = 2^{\ddot{a}_1}5^{\ddot{a}_3}...\alpha^{\ddot{a}_{n+1}}$,$\ddot{a}_2 = 0$. We must also overve that if $a_n \neq 0 \iff \ddot{a}_n = 0$, and incoming argument has no 2 as factor, therefore output has, definitely one or more. Output of step 3 always go next to 2.

4. We repeat 1 until we fall to size of x is zero(bruteforceing keyspace to 0), ie X=1, in case of 2: $size(X) := size(X) - a_1$, in case of 3: $size(X) \cong size(X) + \log_2(3) \approx \frac{3}{2}$, where $a_1 = 1 \vee a_1 \in [2..\infty]$with exactly similar probability.

It concludes proof.

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