

# Collatz conjecture convergence proof

Timo Junolainen

18.3.2020-21.3.2020 Turku university

**Introduction** Collatz conjecture is famous, well known problem. Definition of Collatz conjecture follows:

$$a(n) = \begin{cases} \frac{n}{2} & 2|n \\ 3n + 1 & 2 \nmid n \end{cases}$$

**Lemma.**  $x = p_1 p_2 \dots p_n, p \in \mathbb{P} \rightarrow p_1, p_2 \dots (x + 1)$

**Proof** In the next algorithm I am going to show that collatz sequence always converges to 1.

1. We define input and 'operation' as  $X = 2^{a_1} 3^{a_2} 5^{a_3} \dots$  Sequence of  $a_n$  ends when there is no more factors - size of X ( $size(n) = \log_2(n)$ ) falls to 0. Otherwise we take X as input and derive size of it.
2. X: In case of  $a_1 \neq 0$  - we reduce size by  $a_1$  we also remove factors of 2, ie  $X = 2^{a_1} 3^{a_2} 5^{a_3} \dots \rightarrow X = 2^0 3^{a_2} 5^{a_3} \dots$  Then  $size(X) = \log_2(X) - a_1, a_1 \in [1.. \infty]$ . Output of this step is always producing argument for step 3.
3. X: In case of  $a_1 = 0$  - we do next - we shuffle in  $3^1$  increasing  $size(X) := size(X) + \log_2(3)$ , Then we shuffle all factors, giving in result:  $X = 2^{\ddot{a}_1} 5^{\ddot{a}_3} \dots \alpha^{\ddot{a}_{n+1}}, \ddot{a}_2 = 0$ . We must also overve that if  $a_n \neq 0 \iff \ddot{a}_n = 0$ , and incoming argument has no 2 as factor, therefore output has, definitely one or more. Output has no 3 as factor. Output of step 3 always go next to 2.
4. We repeat 1 until we fall to size of x is zero (bruteforceing keyspace to 0), ie  $X=1$ , in case of 2:  $size(X) := size(X) - a_1$ , in case of 3:  $size(X) \cong size(X) + \log_2(3) \approx \frac{3}{2}$ , where  $a_1 = 1 \vee a_1 \in [2.. \infty]$  with exactly similar probability. Simply said, once size adds 1.5, once removes  $2^n$  where n can be 1 or more.

It concludes proof.

□

revision:0.6b